History based flow analysis in the lambda calculus

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Jean-Jacques Lévy

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Plan

1. Motivations
2. λ-calculus, principals and independence
3. λ-calculus and the Chinese Wall
4. Future works

Motivations

Security and Programming languages

- Restricting rights of downloaded programs is not sufficient...

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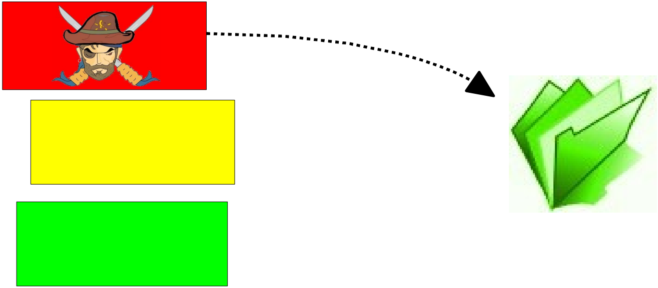

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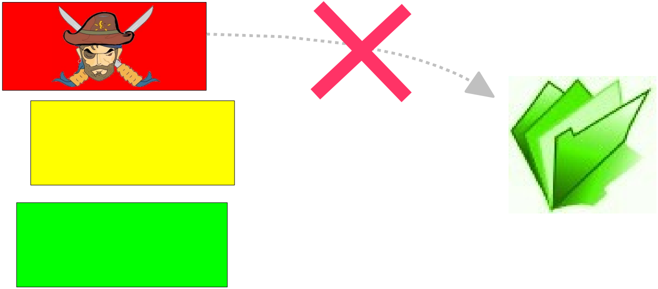

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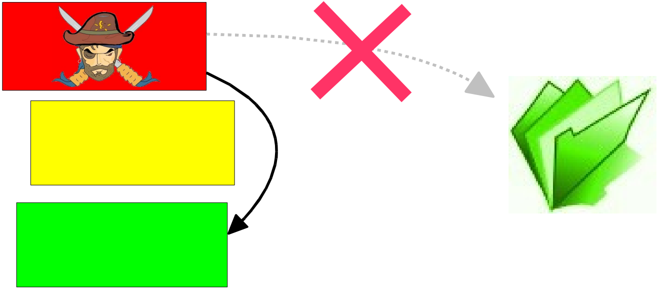
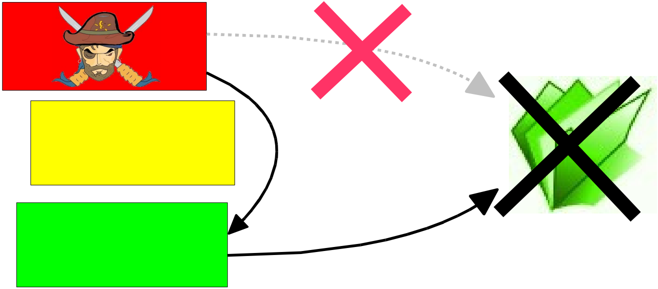


Security and Programming languages

- Restricting rights of downloaded programs is not sufficient...
- ... since attackers can borrow privileges from local programs [Hardy].



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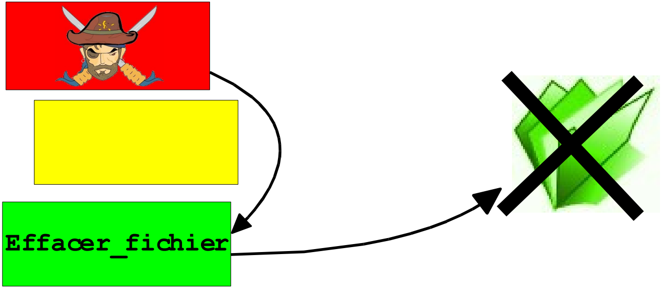

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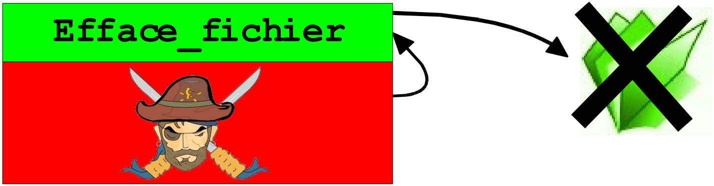

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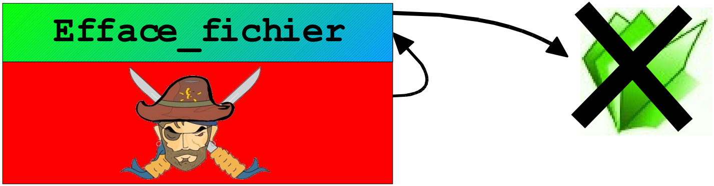

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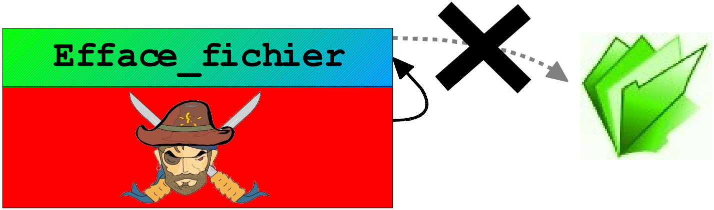

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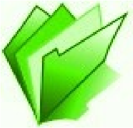

Navigateur

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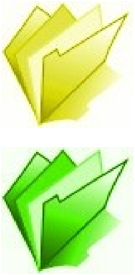

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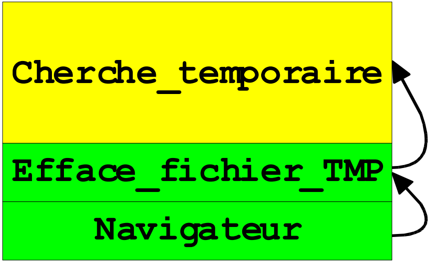

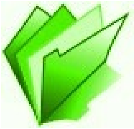

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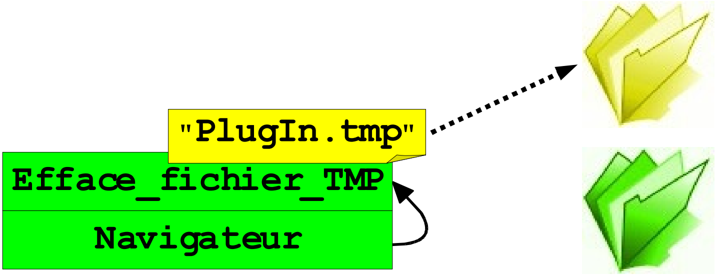

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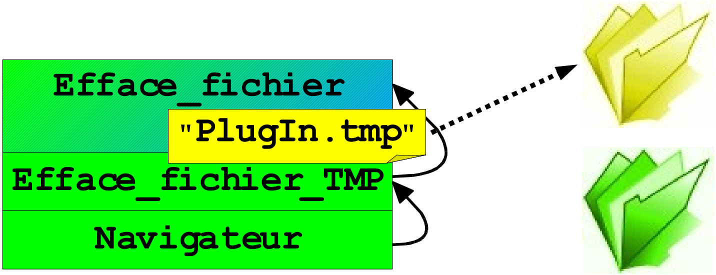

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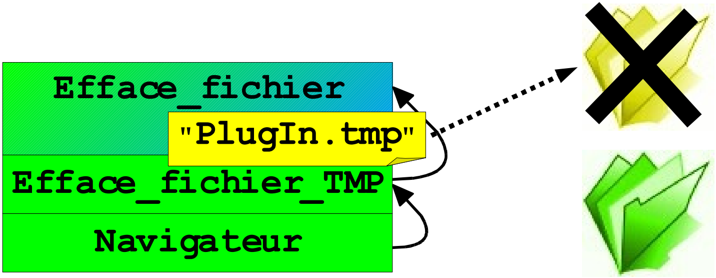

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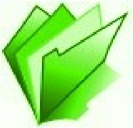

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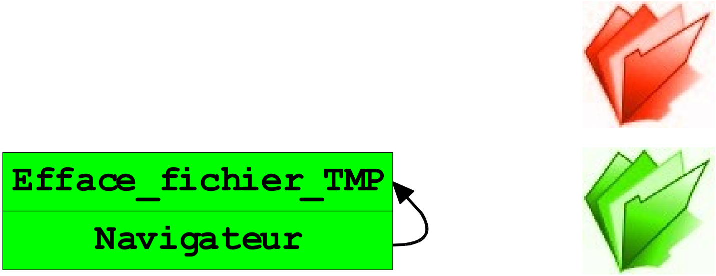

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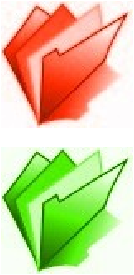

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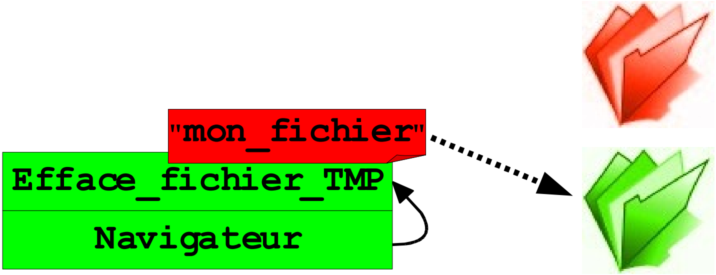

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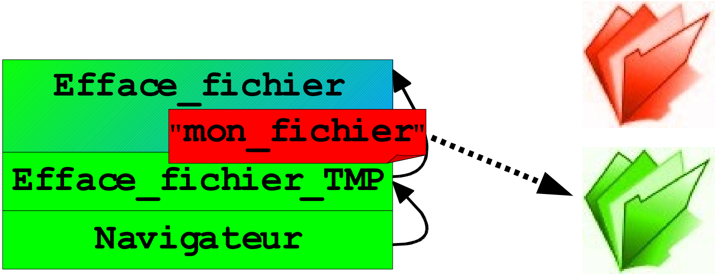

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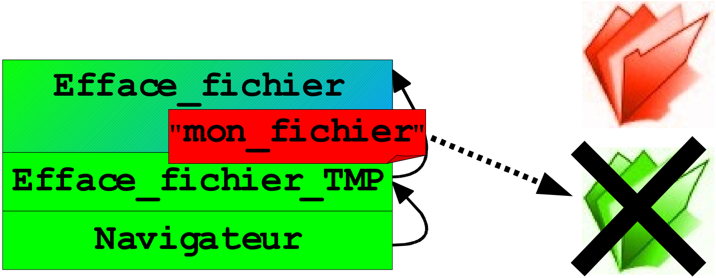

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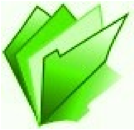

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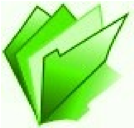

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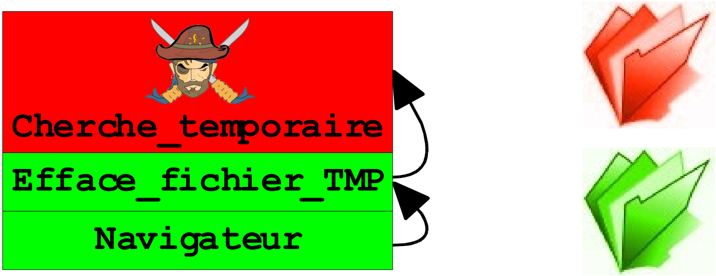

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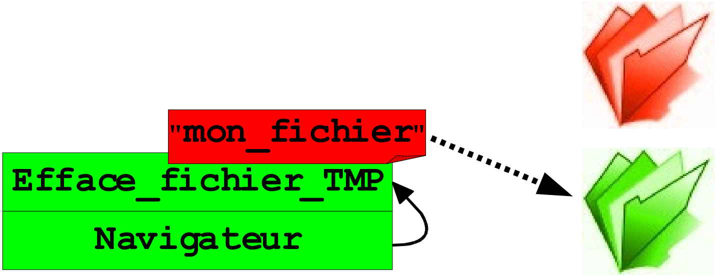

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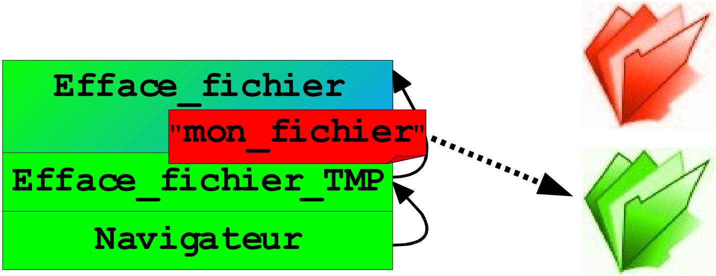

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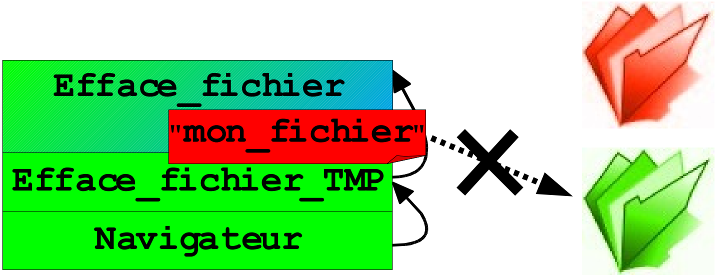

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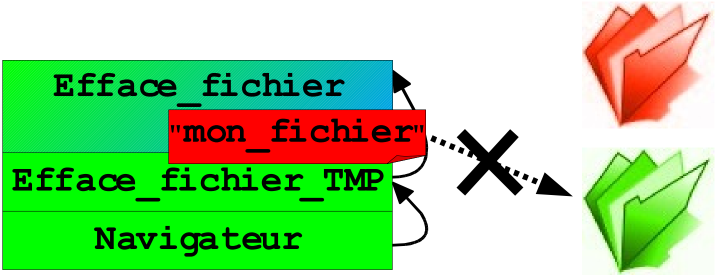

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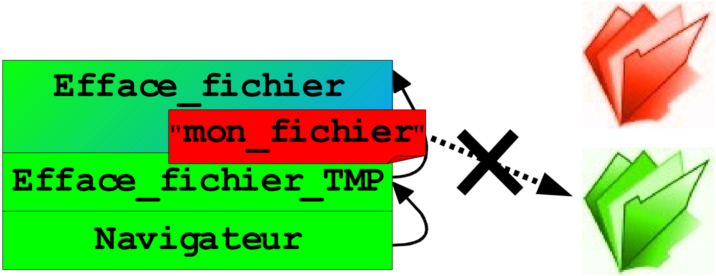
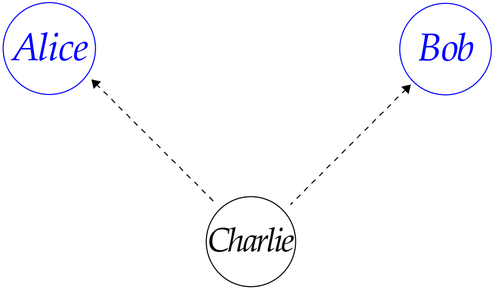
- Static analysis is do-able even on complete languages (FlowCaml, JIF).



Third approach : the Chinese Wall

- Conflicts of interest in « economy » `[Brewer-Nash]`.
- Alice and Bob compete for a contract ; Charlie is the buyer.
- Alice and Bob fix the price of the contract.
- Charlie wants to negotiate the price.



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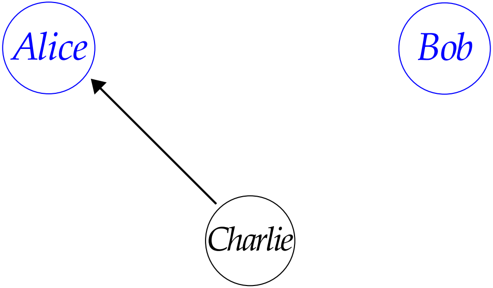

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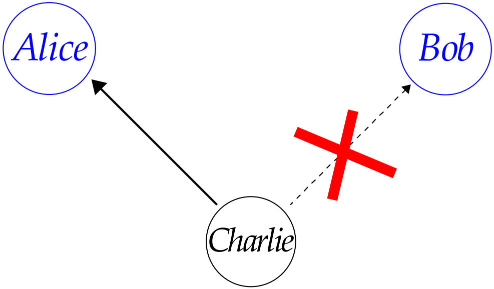

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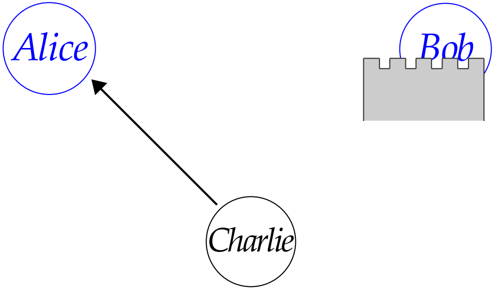

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Summary

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Stack Inspection	-
Flow Information	Non interference
Chinese Wall	**?**

Objectives :

- define the Chinese Wall in the λ-calculus.
- examine the safety property of the Chinese Wall policy.

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λ-calculus, principals and independence

λ_n-calculus : a λ-calculus with principals

- Alice, Bob, Charlie are principals.

$$A, B, \ldots$$

- Terms of λ_n-calculus :

$$\begin{array}{rcll} M,\ N & ::= & x & \textit{Variable} \\ & | & (\lambda x.M)^A & \textit{Abstraction} \\ & | & (MN)^A & \textit{Application} \end{array}$$

- Values :

$$V ::= (\lambda x.M)^A$$

- **Remark** : principals differ from labels in the labelled λ-calculus.

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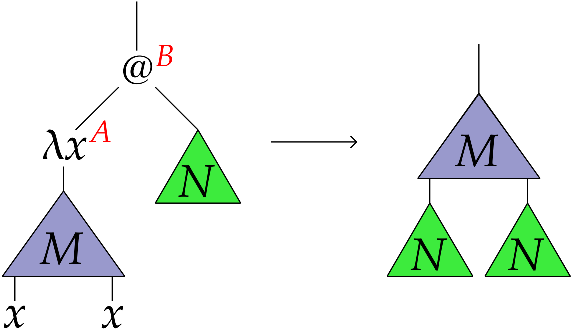
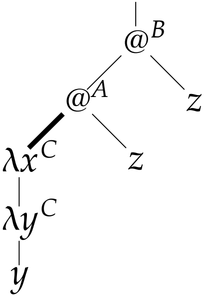
Reduction in λ_n-calculus

$$(\beta) \quad ((\lambda x.M)^{A} N)^{B} \to M\{x \backslash N\}$$



An example of reduction in the λ_n-calculus



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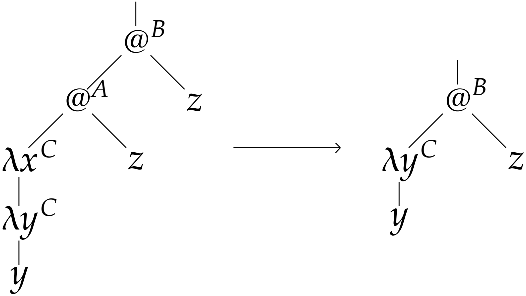

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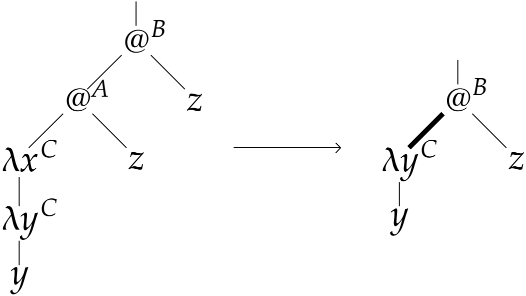

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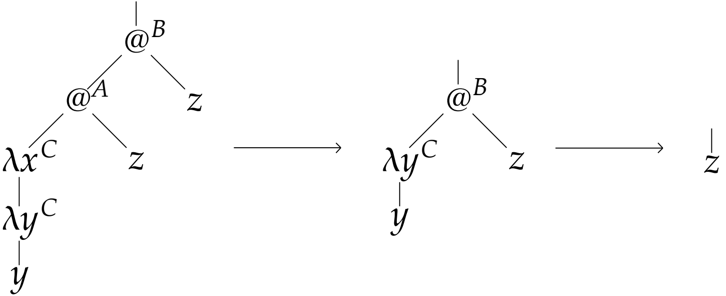

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Basic properties of the λ_n-calculus

- Confluence
- Finite Developments
- Standardisation

Reduction ignoring a principal

Definition

The reduction $M \xrightarrow{((\lambda x.N)^B P)^C} M'$ *ignores* A *iff* $A \notin \{B, C\}$.

- Also written $M \xrightarrow{\urcorner A} M'$.
- We write $M \xrightarrow{\urcorner A}\!\!\!\!\!\!\rightarrow M'$ if every reduction step ignores A.

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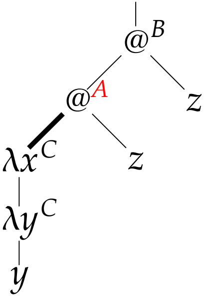

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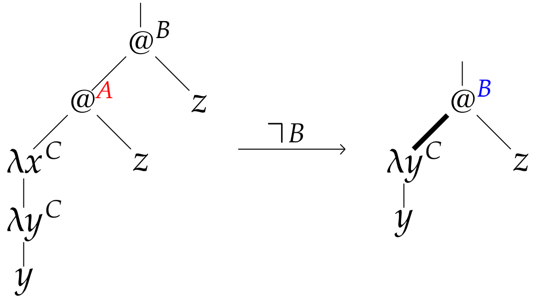

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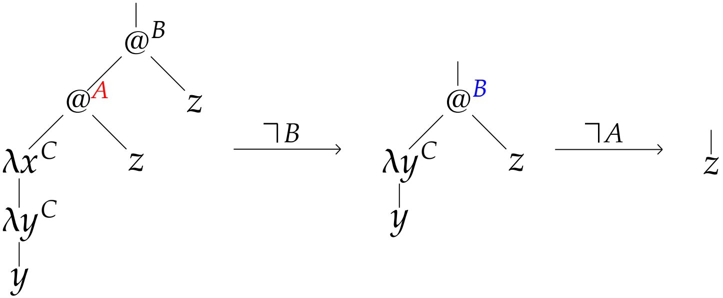

Independence

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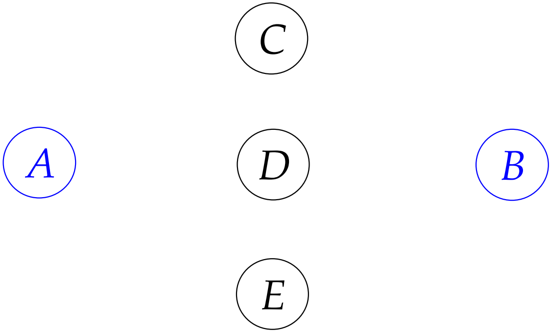

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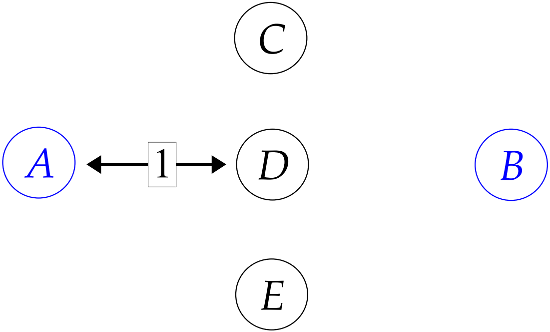

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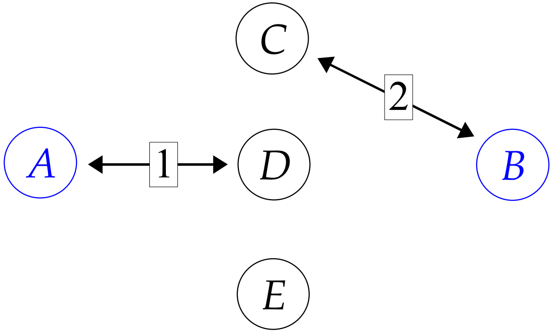

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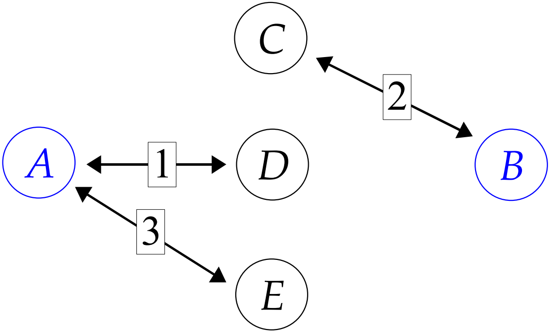

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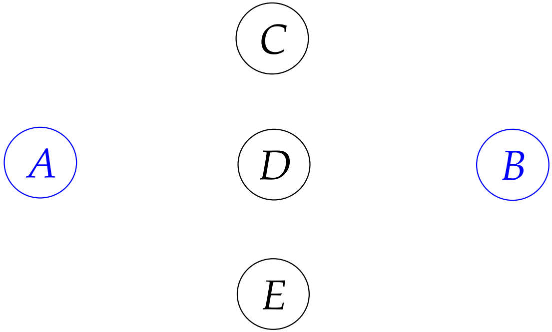

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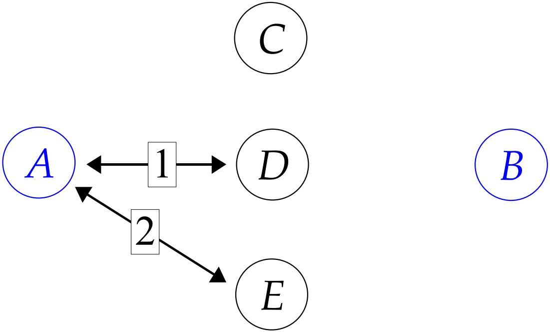

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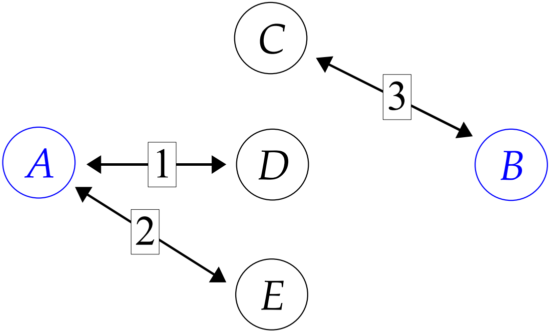

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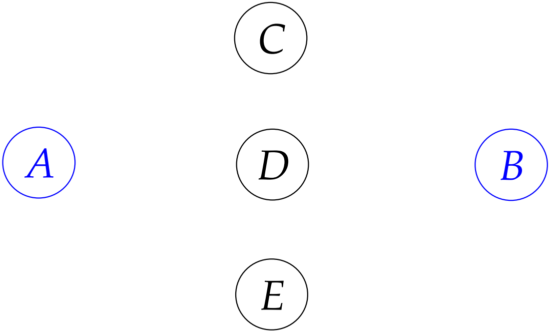

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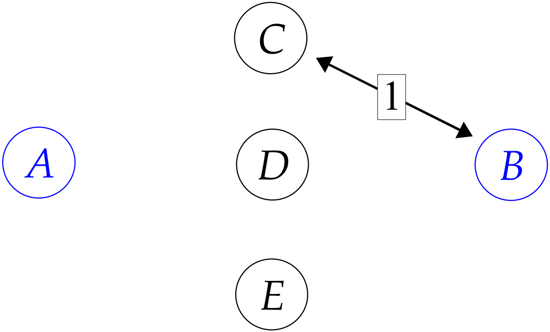

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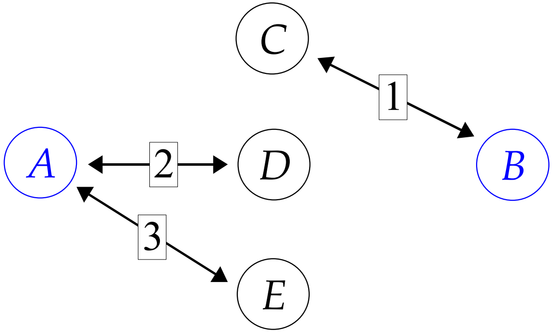

Independence

Definition (Independence)

The reduction $R : M \twoheadrightarrow N$ is independent of the interaction between A and B iff there exists $R_A : M \xrightarrow{\neg A}\mathrel{\mkern-14mu}\twoheadrightarrow M_A$ and $R_B : M \xrightarrow{\neg B}\mathrel{\mkern-14mu}\twoheadrightarrow M_B$ such that $R \leq R'$ (i.e. R/R' is empty) with $R' = R_A;(R_B/R_A) = R_B;(R_A/R_B)$.

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$$
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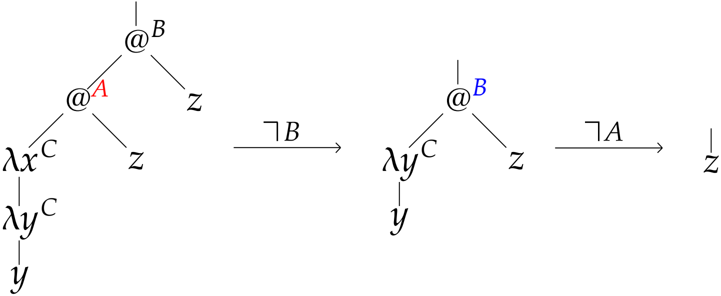
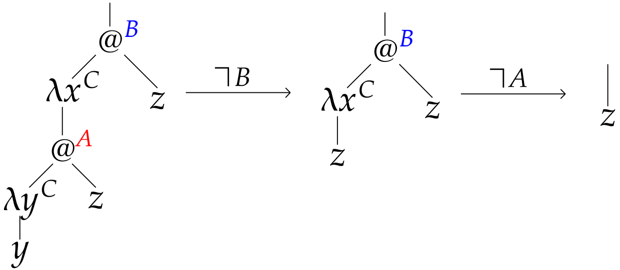
$$

Independence : example 1/2



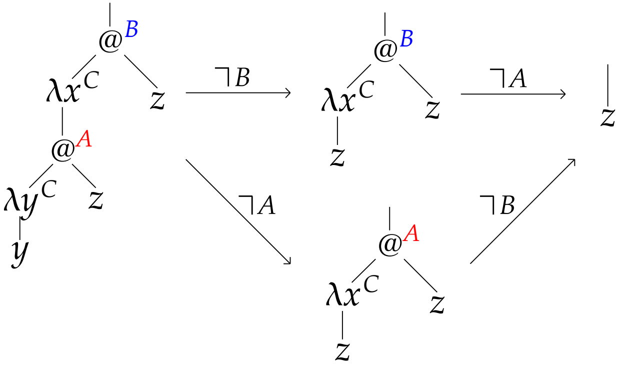
This reduction is not independent of the interaction between A and B.

Independence : example 2/2



This reduction is independent of the interaction between A and B.

Independence : example 2/2



This reduction is independent of the interaction between A and B.

The λ_n-calculus : summary

- A λ-calculus with principals.
- A safety property : independence.
- How to express the Chinese Wall policy in the λ_n-calculus ?
  - This policy relies on history.
  - We use the labelled λ-calculus to track history of interactions.
- Which safety property is guaranteed by the Chinese Wall policy ?
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λ-calculus

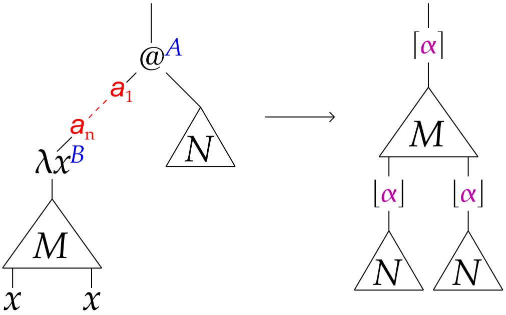
and

the Chinese Wall

The labelled λ_n-calculus

$$
\begin{array}{llcl}
\text{Terms} & M,\ N & ::= & x \\
 & & \mid & (\lambda x.N)^A \\
 & & \mid & (MN)^A \\
 & & \mid & a : M \\
\text{Atomic labels} & a,\ b & ::= & \lceil \alpha \rceil \mid \lfloor \alpha \rfloor \\
\text{Compound labels} & \alpha,\ \beta & ::= & A a_1 a_2 \cdots a_n B \quad n \geq 0 \\
\text{Values} & V,\ W & ::= & (\lambda x.N)^A \mid a : V
\end{array}
$$

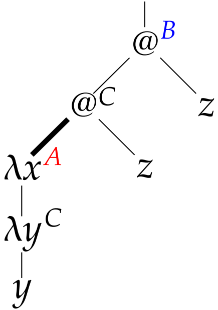
Labelled reduction



$$(\beta) \quad R = (a_1 : \ldots : a_n : (\lambda x.M)^B N)^A \rightarrow \lceil \alpha \rceil : M\{x \backslash \lfloor \alpha \rfloor : N\}$$
$$\alpha = A a_1 \ldots a_n B$$

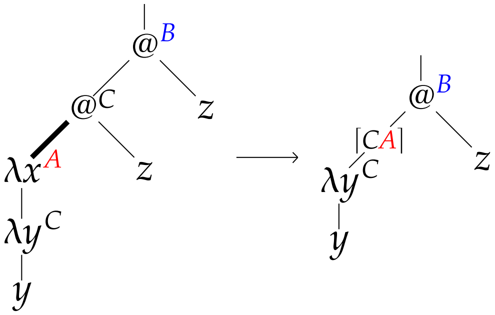
The redex name is $\mathtt{name}(R) = \alpha$.

Labelled reduction : an example



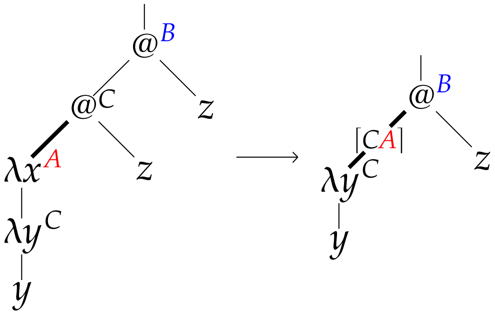
$$(((\lambda x.(\lambda y.y)^C)^A z)^C z)^B$$

Labelled reduction : an example



$$(((\lambda x.(\lambda y.y)^C)^A z)^C z)^B \to (\lceil CA \rceil : (\lambda y.y)^C\ z)^B$$

Labelled reduction : an example



$$(((\lambda x.(\lambda y.y)^C)^A z)^C z)^B \rightarrow ([CA] : (\lambda y.y)^C\ z)^B$$

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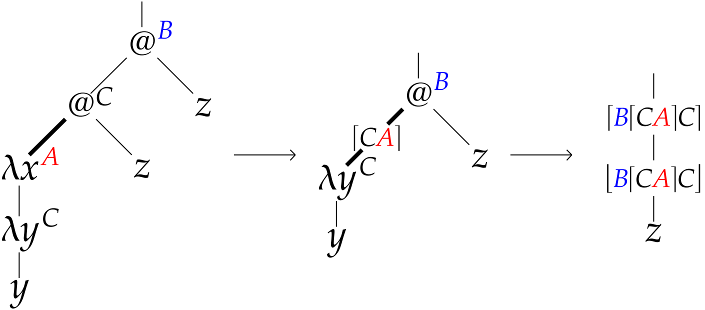

$$
\begin{aligned}
(((\lambda x.(\lambda y.y)^{C})^{A} z)^{C} z)^{B} &\to (\lceil CA \rceil : (\lambda y.y)^{C}\ z)^{B} \\
&\to \lceil B \lceil CA \rceil C \rceil : \lfloor B \lceil CA \rceil C \rfloor : z
\end{aligned}
$$

Independence and labels

- Head sequence : $\tau(x) = \tau((\lambda x.M)^A) = \tau((MN)^A) = 0$
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  - Example : $\tau(a : b : c : (\lambda x.x)^A) = abc$

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- Principals contained in atomic or compound labels :

$$\begin{aligned} \mathtt{Princ}(Aa_1 \dots a_n B) &= \{A, B\} \cup_{1 \le i \le n} \mathtt{Princ}(a_i) \\ \mathtt{Princ}(\lceil \alpha \rceil) &= \mathtt{Princ}(\lfloor \alpha \rfloor) = \mathtt{Princ}(\alpha) \end{aligned}$$

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  - Example : $\mathtt{Princ}(A\lceil B \lfloor AC \rfloor D \rceil E) = \{A, B, C, D, E\}$

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A sequence of atomic labels $a_1 \ldots a_n$ separates the principals A and B iff, for every $1 \le i \le n$, we have $\{A, B\} \not\subseteq \mathtt{Princ}(a_i)$.

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- Examples :
  - ⋆ $\lceil AC \rceil \lfloor C \lceil DE \rceil B \rfloor$ separates A et B.
  - ⋆ $\lceil DC \rceil \lfloor C \lceil AE \rceil B \rfloor$ does not separate A et B.

Independence and labels : separation

Theorem (Separation)

If M is an unlabelled term and if the reduction $M \twoheadrightarrow V$ is independent of the interaction between A and B, then $\tau(V)$ separates A and B.

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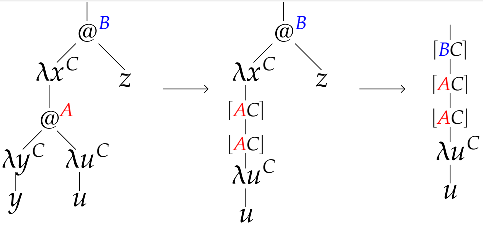

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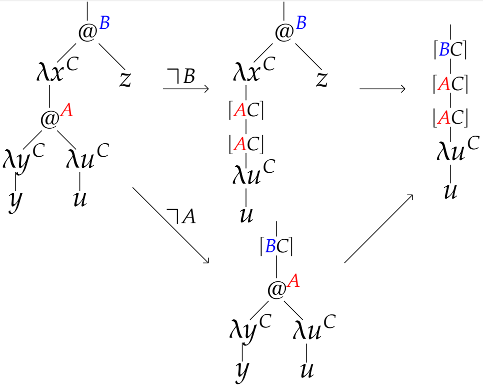

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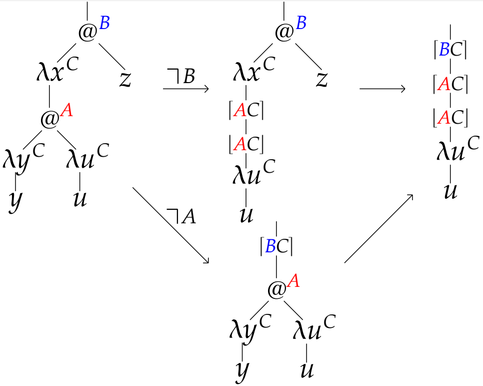

The head sequence $\lceil B\,C \rceil \lceil A\,C \rceil \lfloor A\,C \rfloor$ separates A and B.

Independence and separation

Theorem

If $M \twoheadrightarrow V$ *and if* $\tau(V)$ *separates* A *and* B*, then there is a reduction* $\mathcal{R} : M \twoheadrightarrow W$ *independent of the interaction between* A *and* B*.*

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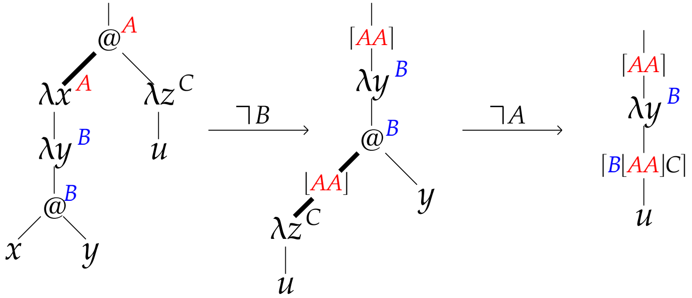

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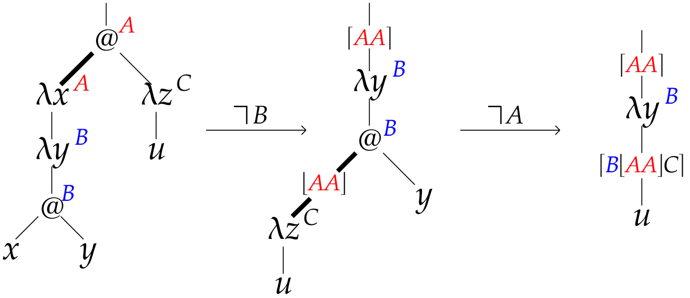

- ⋆ The label $\lceil AA \rceil$ separates A et B.

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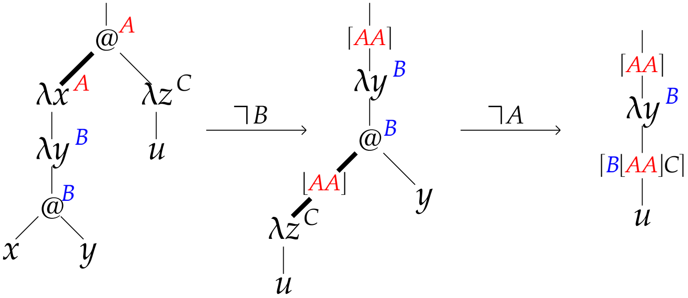

- The label $\lceil AA \rceil$ separates A et B.
- This reduction **is not** independent of the interaction between A and B.

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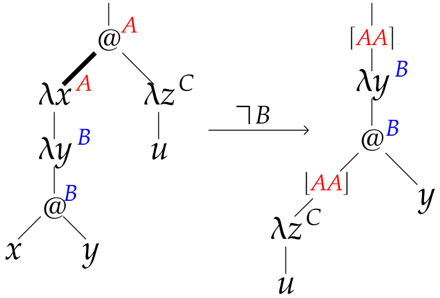

- ⋆ The label $\lceil AA \rceil$ separates A et B.
- ⋆ This reduction **is** independent of the interaction between A and B.

Expressing Chinese Wall in the λ_n-calculus

- The Chinese Wall between A and B is written $\mathcal{CW}(A, B)$.
- If redex R has name $Aa_1 \ldots a_n B$, then :
  - A and B interact (directly).
  - If $C \in \texttt{Princ}(a_i)$, then C participated to the creation of this interaction.

Definition (Chinese Wall)

A reduction follows $\mathcal{CW}(A, B)$ iff every redex R contracted by this reduction is such that :

$$\{A, B\} \not\subseteq \texttt{Princ}(\texttt{name}(R))$$

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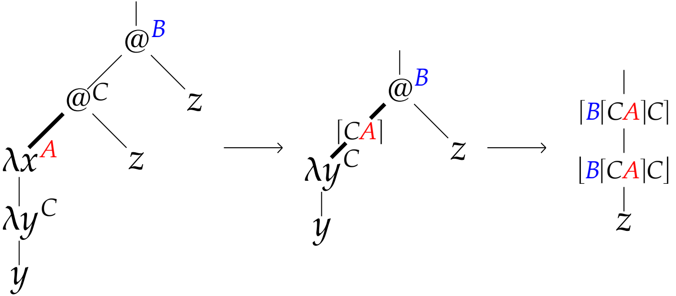
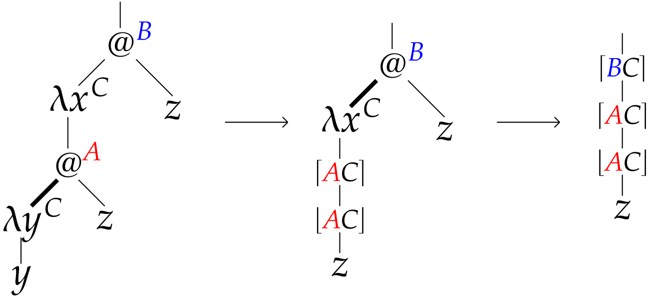


$$\begin{aligned} \texttt{Princ}(\texttt{name}(R_1)) &= \{A, C\} \\ \texttt{Princ}(\texttt{name}(R_2)) &= \{A, B, C\} \end{aligned}$$

This reduction does not follow $\mathcal{CW}(A, B)$.

Chinese Wall in the λ_n-calculus : example 2/2



$$\begin{aligned}\texttt{Princ}(\texttt{name}(R_1)) &= \{A, C\}\\ \texttt{Princ}(\texttt{name}(R_2)) &= \{B, C\}\end{aligned}$$

This reduction follows $\mathcal{CW}(A, B)$.

Correction of $\mathcal{CW}(A, B)$

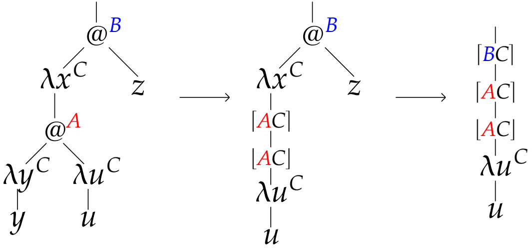
Theorem (Correction)

If $R : M \twoheadrightarrow N$ follows $\mathcal{CW}(A, B)$, then R is independent of the interaction between A and B.

$$
\begin{tikzcd}
 & M \arrow[dl, dotted, two heads, "R_A"'] \arrow[d, two heads, "R"'] \arrow[dr, dotted, two heads, "R_B"] \arrow[dl, phantom, "\neg A"', shift right=4] \arrow[dr, phantom, "\neg B", shift left=4] & \\
M_A \arrow[dr, dotted, two heads, "R_B/R_A"'] & N \arrow[d, dotted, two heads, "R'/R"'] & M_B \arrow[dl, dotted, two heads, "R_A/R_B"] \\
 & N' &
\end{tikzcd}
$$

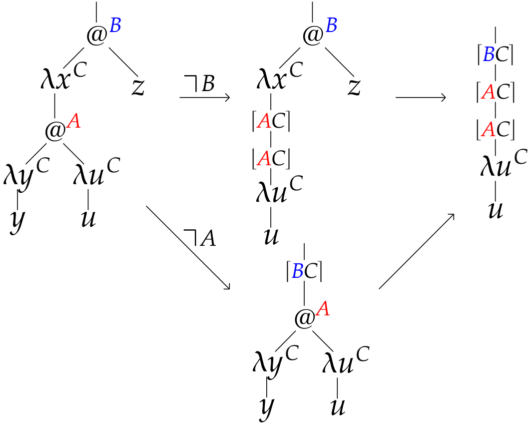
The Chinese Wall guarantees the independence.

Correction of $\mathcal{CW}(A, B)$: example



The reduction follows $\mathcal{CW}(A, B)$...

Correction of $\mathcal{CW}(A, B)$: example



...hence it is independent of the interaction between A and B

Correction of $\mathcal{CW}(A, B)$: proof

- Sublabel of a compound label :

$$
\begin{aligned}
\alpha &\preceq \alpha \\
\alpha &\preceq Aa_1 \ldots a_n B \text{ si } \exists i \;.\; a_i = \lceil \beta \rceil \text{ and } \alpha \preceq \beta \\
\alpha &\preceq Aa_1 \ldots a_n B \text{ si } \exists i \;.\; a_i = \lfloor \beta \rfloor \text{ and } \alpha \preceq \beta
\end{aligned}
$$

- Example : $\alpha \preceq A\lceil\alpha\rceil\lfloor\gamma\rfloor B$

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Correction of $\mathcal{CW}(A, B)$: proof

$$M \xrightarrow{S_1} \xrightarrow{S_2} \xrightarrow{S_3} \cdots \xrightarrow{S_n} N$$

$$R$$

Correction of $\mathcal{CW}(A, B)$: proof

$$M \xrightarrow{S_1} \xrightarrow{S_2} \underset{R}{\xrightarrow{S_3}} \cdots \xrightarrow{S_n} N$$

For $1 \leq i \leq n$, we write $\alpha_i = \mathtt{name}(S_i)$.
We have $\{A, B\} \cap \mathtt{Princ}(\alpha_i) \neq \{A, B\}$.

Correction of $\mathcal{CW}(A, B)$: proof

$$
\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\alpha_1"'] & {} \arrow[r, "S_2"] & {} \arrow[r, "S_3"] & R \arrow[r, dashed] & {} \arrow[r, "S_n"] & N \\
{} \arrow[d, Rightarrow, "\alpha_2"'] & & & & & \\
{\color{blue}R_1} \quad {} \arrow[d, Rightarrow, "\alpha_3"'] & & & & & \\
{} \arrow[d, Rightarrow, dashed] & & & & & \\
{} \arrow[d, Rightarrow, "\alpha_n"'] & & & & & \\
N_1 & & & & &
\end{tikzcd}
$$

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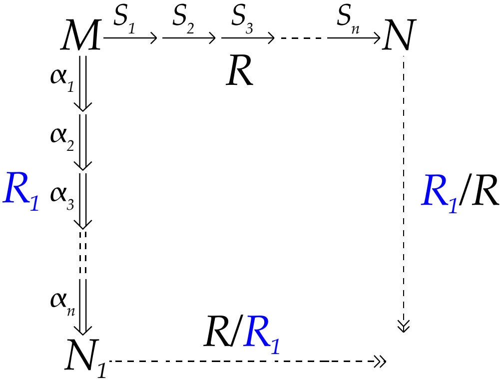
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\alpha_1"'] & \cdot \arrow[r, "S_2"] & \cdot \arrow[r, "S_3"] \arrow[r, phantom, "R"' below] & \cdot \arrow[r, dashed] & \cdot \arrow[r, "S_n"] & N \arrow[dddd, dashed, ->>, "R_1/R"] \\
\cdot \arrow[d, Rightarrow, "\alpha_2"'] & & & & & \\
\cdot \arrow[d, Rightarrow, "\alpha_3"', "R_1"' near start] & & & & & \\
\cdot \arrow[d, Rightarrow, dashed] & & & & & \\
\cdot \arrow[d, Rightarrow, "\alpha_n"'] & & & & & \cdot \\
N_1 \arrow[rrrrru, dashed, ->>, "R/R_1"] & & & & &
\end{tikzcd}
$$

For $1 \leq i \leq n$, we write $\alpha_i = \texttt{name}(S_i)$.
We have $\{A, B\} \cap \texttt{Princ}(\alpha_i) \neq \{A, B\}$.

Correction of $\mathcal{CW}(A, B)$: proof



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Lemma (Completion)

If $R : M \xrightarrow{S_1} \dots \xrightarrow{S_n} N$ and if for every i, we have $\mathtt{name}(S_i) = \alpha_i$, then $R_1 : M \xRightarrow{\alpha_1} \dots \xRightarrow{\alpha_n} N_1$ and $R \leq R_1$.

Correction of $\mathcal{CW}(A, B)$: proof

$$
\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\alpha_1"'] & {} \arrow[r, "S_2"] & {} \arrow[r, "S_3", "R"'] & {} \arrow[r, dashed, no head] & {} \arrow[r, "S_n"] & N \arrow[ddddd, dashed, two heads, "R_1/R"] \\
{} \arrow[d, Rightarrow, "\alpha_2"'] & & & & & \\
{} \arrow[d, Rightarrow, "R_1 \quad \alpha_3"'] & & & & & \\
{} \arrow[d, dashed, Rightarrow, no head] & & & & & \\
{} \arrow[d, Rightarrow, "\alpha_n"'] & & & & & \\
N_1 \arrow[rrrrr, dashed, two heads, "R/R_1 = \cancel{0}^n"] & & & & & N_1
\end{tikzcd}
$$

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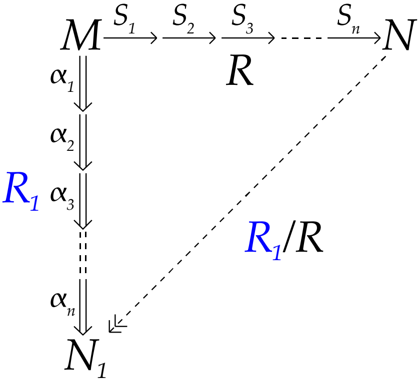

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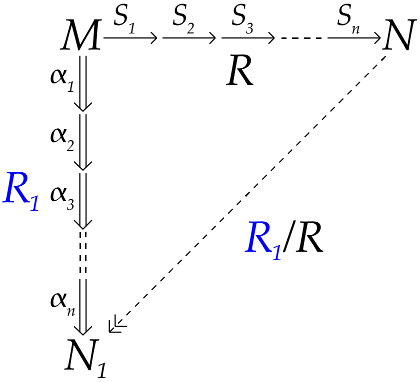
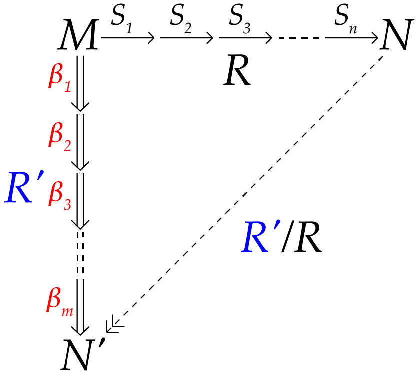


For $1 \leq i \leq n$, we write $\alpha_i = \texttt{name}(S_i)$.
We have $\{A, B\} \cap \texttt{Princ}(\alpha_i) \neq \{A, B\}$.

Lemma (Reordering)

If $R : M \overset{\alpha_1}{\Longrightarrow} \ldots \overset{\alpha_n}{\Longrightarrow} N$, there is a reduction $R' : M \overset{\beta_1}{\Longrightarrow} \ldots \overset{\beta_m}{\Longrightarrow} N'$ such that
(1) $\{\beta_i\}_{1 \leq i \leq m} \subseteq \{\alpha_i\}_{1 \leq i \leq n}$ (2) if $i < j$, then $\beta_j \not< \beta_i$ (3) $R \leq R'$

Correction of $\mathcal{CW}(A, B)$: proof



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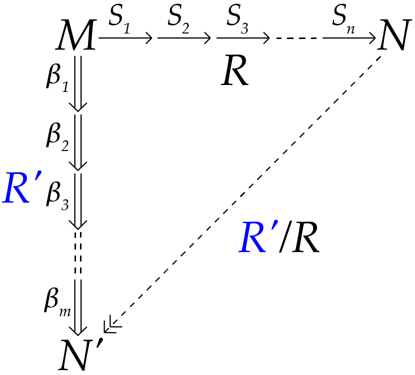
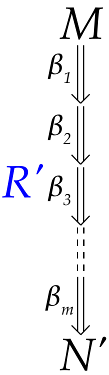
Correction of $\mathcal{CW}(A, B)$: proof

$$\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\beta_1"'] & \cdot \arrow[r, "S_2"] & \cdot \arrow[r, "S_3"] & \cdot \arrow[r, dashed] & \cdot \arrow[r, "S_n"] & N \arrow[dddddlllll, dashed, "{\color{blue}R'}/R"] \\
\cdot \arrow[d, Rightarrow, "\beta_2"'] & & R & & & \\
\cdot \arrow[d, Rightarrow, "{\color{blue}R'}\ \beta_3"'] & & & & & \\
\cdot \arrow[d, Rightarrow, dashed] & & & & & \\
\cdot \arrow[d, Rightarrow, "\beta_m"'] & & & & & \\
N' & & & & &
\end{tikzcd}
\qquad
\begin{tikzcd}
M \arrow[d, Rightarrow, "\beta_1"'] \\
\cdot \arrow[d, Rightarrow, "\beta_2"'] \\
\cdot \arrow[d, Rightarrow, "{\color{blue}R'}\ \beta_3"'] \\
\cdot \arrow[d, Rightarrow, dashed] \\
\cdot \arrow[d, Rightarrow, "\beta_m"'] \\
N'
\end{tikzcd}$$

- If $i < j$, we have $\beta_j \not\prec \beta_i$.
- $\{\gamma_i\}_{1\leq i\leq k}$: elements of $\{\beta_i\}_{1\leq i\leq m}$ such that $\{A, B\} \cap \mathtt{Princ}(\beta_i) = \emptyset$.
- $\{\delta_i\}_{1\leq i\leq k'}$: elements of $\{\beta_i\}_{1\leq i\leq m}$ such that $\{A, B\} \cap \mathtt{Princ}(\beta_i) \neq \emptyset$.
- If $\beta_i \in \{\delta_i\}_{1\leq i\leq k'}$, if $\beta_j \in \{\gamma_i\}_{1\leq i\leq k}$, we have $\beta_i \not\prec \beta_j$.

Correction of $\mathcal{CW}(A, B)$: proof





- If $i < j$, we have $\beta_j \not\leftarrow \beta_i$.
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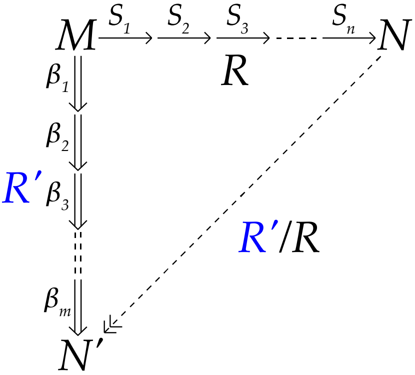
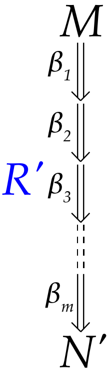
- If $\beta_i \in \{\delta_i\}_{1 \leq i \leq k'}$, if $\beta_j \in \{\gamma_i\}_{1 \leq i \leq k}$, we have $\beta_i \not\leftarrow \beta_j$

Correction of $\mathcal{CW}(A, B)$: proof





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Correction of $\mathcal{CW}(A, B)$: proof

$$
\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\beta_1"'] & {} \arrow[r, "S_2"] & {} \arrow[r, "S_3", "R"'] & {} \arrow[r, dashed, no head] & {} \arrow[r, "S_n"] & N \arrow[dddddlllll, dashed, Rightarrow, "{\color{blue}R'}/R"] \\
{} \arrow[d, Rightarrow, "\beta_2"'] & & & & & \\
{} \arrow[d, Rightarrow, "{\color{blue}R'}\ \beta_3"'] & & & & & \\
{} \arrow[d, dashed, Rightarrow, no head] & & & & & \\
{} \arrow[d, Rightarrow, "\beta_m"'] & & & & & \\
N' & & & & &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
M \arrow[d, Rightarrow, "\beta_1"'] \\
{} \arrow[d, Rightarrow, "\beta_2"'] \\
{} \arrow[d, Rightarrow, "{\color{blue}R'}\ \beta_3"'] \\
{} \arrow[d, dashed, Rightarrow, no head] \\
{} \arrow[d, Rightarrow, "\beta_m"'] \\
N'
\end{tikzcd}
$$

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Correction of $\mathcal{CW}(A, B)$: proof

$$
\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\beta_1"'] & \cdot \arrow[r, "S_2"] & \cdot \arrow[r, "S_3", "R"'] & \cdot \arrow[r, dashed, no head] & \cdot \arrow[r, "S_n"] & N \arrow[ddddlllll, dashed, twoheadrightarrow, "R'/R"] \\
\cdot \arrow[d, Rightarrow, "\beta_2"'] \\
\cdot \arrow[d, Rightarrow, "R'\ \beta_3"'] \\
\cdot \arrow[d, Rightarrow, dashed] \\
\cdot \arrow[d, Rightarrow, "\beta_m"'] \\
N'
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\qquad
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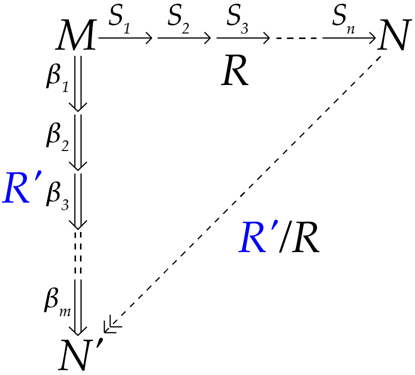
\cdot \arrow[d, Rightarrow, "R'\ \beta_3"'] \\
\cdot \arrow[d, Rightarrow, dashed] \\
\cdot \arrow[d, Rightarrow, "\beta_m"'] \\
N'
\end{tikzcd}
$$

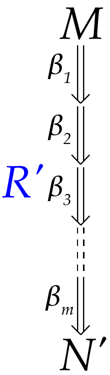
- If $i < j$ and $\beta_i \in \{\delta_i\}_{1 \leq i \leq k'}$ and $\beta_j \in \{\gamma_i\}_{1 \leq i \leq k}$, we have $\beta_i \not\prec \beta_j$ and $\beta_j \not\prec \beta_i$.

Lemma (Permutation)

If $\alpha \not\prec \beta$ and $\beta \not\prec \alpha$ and if $R_1 : M \overset{\alpha}{\Rightarrow}\overset{\beta}{\Rightarrow} N$, then we have $R_2 : M \overset{\beta}{\Rightarrow}\overset{\alpha}{\Rightarrow} N$ and $R_1 \sim R_2$.

Correction of $\mathcal{CW}(A, B)$: proof





- If $i < j$ and $\beta_i \in \{\delta_i\}_{1 \leq i \leq k'}$ and $\beta_j \in \{\gamma_i\}_{1 \leq i \leq k}$, we have $\beta_i \not\prec \beta_j$ and $\beta_j \not\prec \beta_i$.

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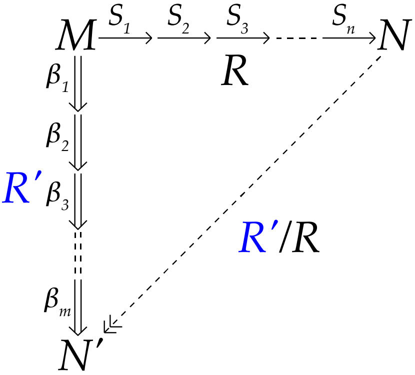

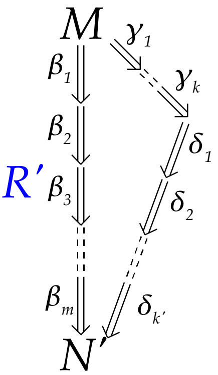

- If $i < j$ et $\beta_i \in \{\delta_i\}_{1 \leq i \leq k'}$ and $\beta_j \in \{\gamma_i\}_{1 \leq i \leq k}$, we have $\beta_i \not\prec \beta_j$ et $\beta_j \not\prec \beta_i$.

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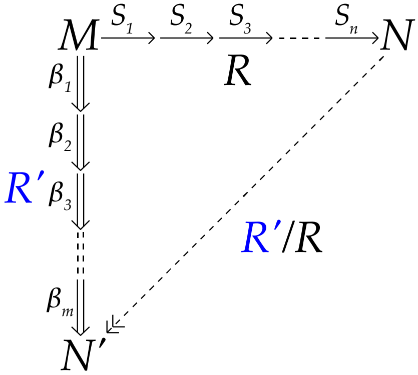
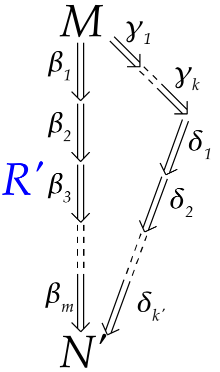
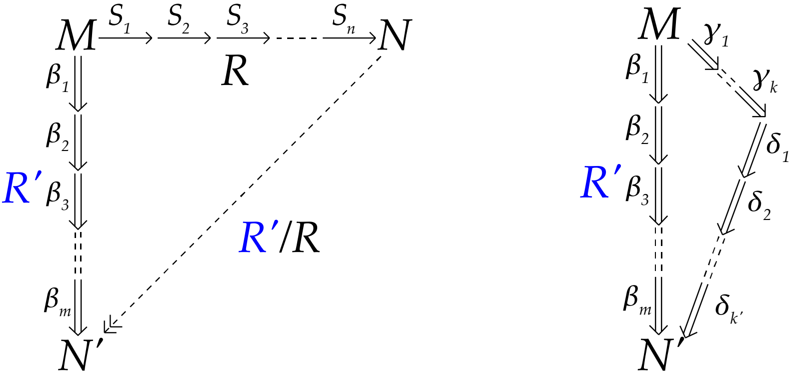
Correction of $\mathcal{CW}(A, B)$: proof





- $\{\eta_i\}_{1 \leq i \leq p}$: elements of $\{\delta_i\}_i$ such that $\{A, B\} \cap \texttt{Princ}(\delta_i) = \{A\}$.
- $\{\theta_i\}_{1 \leq i \leq p'}$: elements of $\{\delta_i\}_i$ such that $\{A, B\} \cap \texttt{Princ}(\delta_i) = \{B\}$.
- For every i, j, we have $\eta_i \not< \theta_j$ and $\theta_j \not< \eta_i$.

Correction of $\mathcal{CW}(A, B)$: proof



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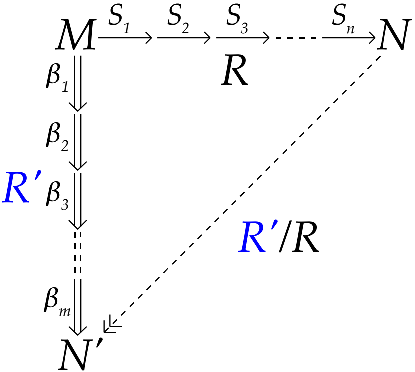

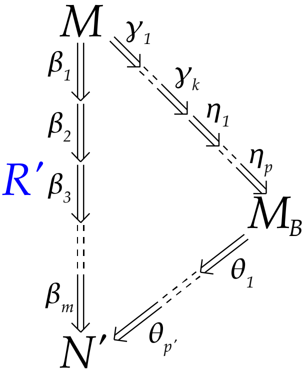

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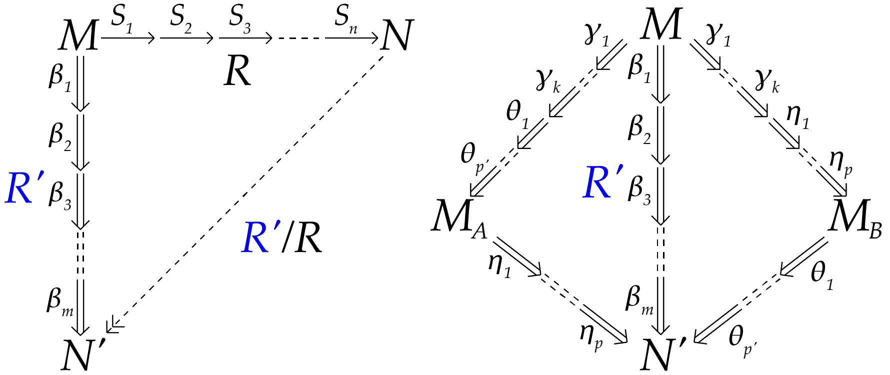

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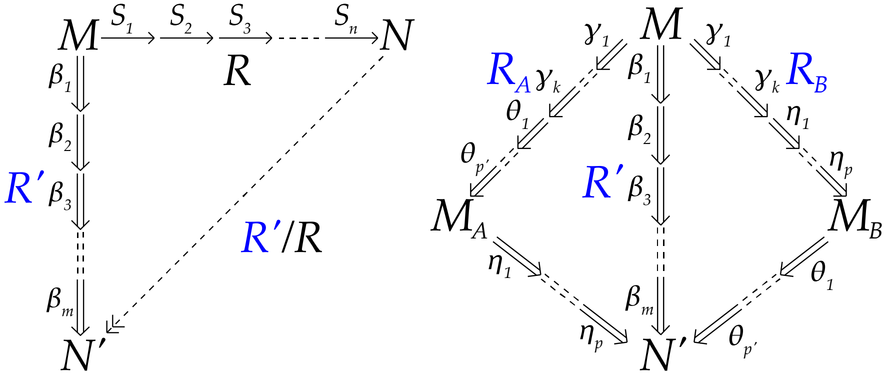
Correction of $\mathcal{CW}(A, B)$: proof



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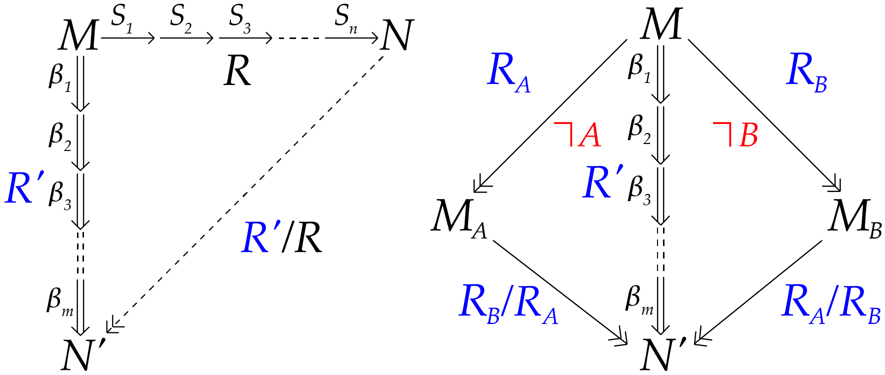


Correction of $\mathcal{CW}(A, B)$: proof

$$
\begin{tikzcd}
M \arrow[r, "S_1"] \arrow[d, Rightarrow, "\beta_1"'] & \cdot \arrow[r, "S_2"] & \cdot \arrow[r, "S_3"] \arrow[d, phantom, "R"] & \cdot \arrow[r, dashed, "S_n"] & N \arrow[dddddllll, dashed, "R'/R"] \\
\cdot \arrow[d, Rightarrow, "\beta_2"'] & & & & \\
\cdot \arrow[d, Rightarrow, "\beta_3"', "R'"'{xshift=-2em}] & & & & \\
\cdot \arrow[d, Rightarrow, dashed] & & & & \\
\cdot \arrow[d, Rightarrow, "\beta_m"'] & & & & \\
N' & & & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & M \arrow[dl, "R_A"'] \arrow[dr, "R_B"] \arrow[dd, Rightarrow, "\beta_1 \, \beta_2 \, \beta_3 \cdots \beta_m", "R'"'] & \\
M_A \arrow[r, phantom, "\neg A"] \arrow[dr, "R_B/R_A"'] & {} \arrow[r, phantom, "\neg B"] & M_B \arrow[dl, "R_A/R_B"] \\
 & N' &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & M \arrow[dl, dotted, "R_A"'] \arrow[d, "R"] \arrow[dr, dotted, "R_B"] & \\
M_A \arrow[dr, dotted, "R_B/R_A"'] \arrow[ur, phantom, "\neg A"{xshift=2em}] & N \arrow[d, dotted, "R'/R"'] & M_B \arrow[dl, dotted, "R_A/R_B"] \arrow[ul, phantom, "\neg B"{xshift=-2em}] \\
 & N' &
\end{tikzcd}
$$

λ-calculus and Chinese Wall : summary

1. Safety property : independence
2. Correspondence between labelled lambda calculus and independence

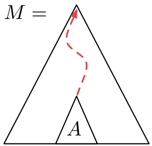
Safety policy	*Safety property*
Stack inspection	-
Information flow	Non interference
Chinese Wall	Independence

Future works

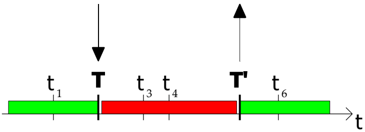
Objectives

1. Static information flow in the λ-calculus
   - labelled λ-calculus and DCC `[Riecke]`, FlowCaml as `[Simonet, Pottier]`, DCC+ `[Abadi]`, `etc`
2. Reduction strategies
   - call-by-value λ-calculus
   - weak λ-calculus
3. Adding delta rules
   - Imperative features and exceptions
   - Safety rules (safety operators : `uses` or `binds`)
4. Concurrent features
   - Permutation equivalence and Event structures
   - Reversible processes (backtracking) `[Jean Krivine]`

Conclusion : non interference

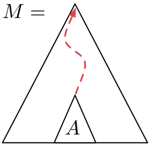


- Non interference : the labels of the λ-calculus express functional interference.
- In the λ-calculus with references, labels have to also capture interference with memory.
  - A memory cell interferes within some time interval.



  - We can use irreversibility of contexts in the labelled λ-calculus [Blanc].

Conclusion : non interference



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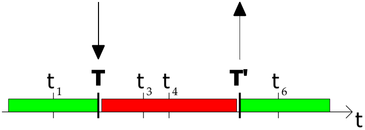

  - We can use irreversibility of contexts in the labelled λ-calculus `[Blanc]`.

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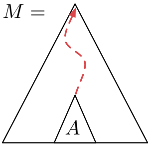

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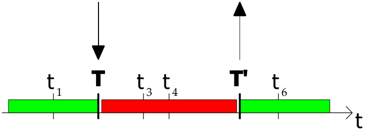
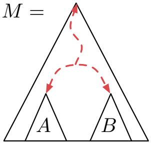
- In the λ-calculus with references, labels have to also capture interference with memory.
  - A memory cell interferes within some time interval.



  - We can use irreversibility of contexts in the labelled λ-calculus [Blanc].

Conclusion : independence



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2. Link between non-interference and independence : express these properties within a common framework.
3. Dynamic labels are a good starting point for an analysis mixing static and dynamic checks.
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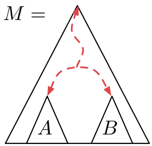

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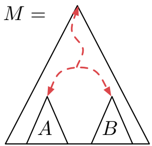

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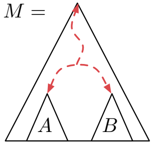

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